

7 Measurement Geometries

Light Measurement Handbook
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International Light Technologies, Inc.

Solid Angles

One of the key concepts to understanding the relationships between measurement geometries is that of the solid angle, or steradian. A sphere contains 4π steradians. A solid angle which, having its vertex at the center of the sphere, cuts off a spherical surface area equal to the square of the radius of the sphere. For example, a one steradian section of a one meter radius spherical surface subtends a surface area of one square meter.

The sphere shown in cross section in figure 7.1 illustrates the concept. A solid angle of one steradian has been removed from the sphere. This removed cone is shown in figure 7.2. The solid angle, Ω , in steradians, is equal to the spherical surface area, A , divided by the square of the radius, r .

Most radiometric measurements do not require an accurate calculation of the spherical surface area to convert between units. Flat area estimates can be substituted for spherical area when the solid angle is less than 0.03 steradians, resulting in an error of less than one percent. This roughly translates to a distance at least 5 times greater than the largest dimension of the detector. In general, if you follow the “five times rule” for approximating a point source (see Chapter 6), you can safely estimate using planar surface area.

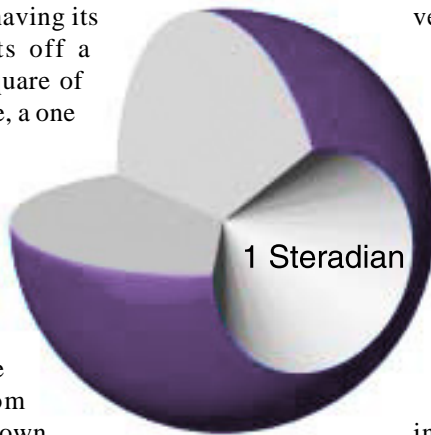


Fig. 7.1

shown in cross section in figure 7.2. The solid angle, Ω , in steradians, is equal to the spherical surface area, A , divided by the square of the radius, r .

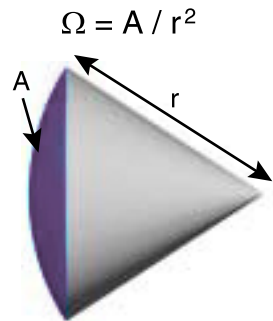


Fig. 7.2 Solid angle

Radiant and Luminous Flux

Radiant flux is a measure of radiometric power. Flux, expressed in watts, is a measure of the rate of energy flow, in joules per second. Since photon energy is inversely proportional to wavelength, ultraviolet photons are more powerful than visible or infrared.

Luminous flux is a measure of the power of visible light. Photopic flux, expressed in lumens, is weighted to match the responsivity of the human eye, which is most sensitive to yellow-green.

Scotopic flux is weighted to the sensitivity of the human eye in the dark adapted state.

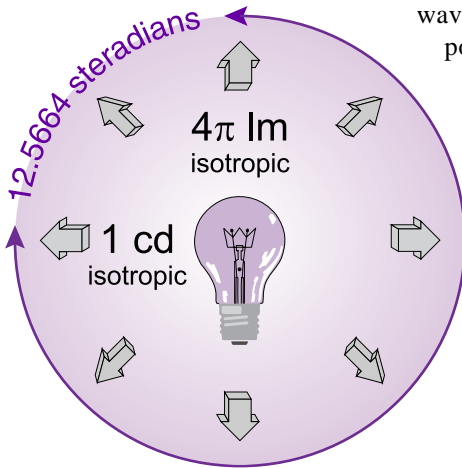


Fig. 7.3 total flux output.

Units Conversion: Power

RADIANT FLUX:

- 1 W (watt)
 - = 683.0 lm at 555 nm
 - = 1700.0 scotopic lm at 507 nm
- 1 J (joule)
 - = 1 W*s (watt * second)
 - = 107 erg
 - = 0.2388 gram * calories

LUMINOUS FLUX:

- 1 lm (lumen)
 - = 1.464×10^{-3} W at 555 nm
 - = $1/(4\pi)$ candela (only if isotropic)
- 1 lm*s (lumen * seconds)
 - = 1 talbot (T)
 - = 1.464×10^{-3} joules at 555 nm

λ nm	Photopic Luminous Efficiency	Photopic lm / W Conversion	Scotopic Luminous Efficiency	Scotopic lm / W Conversion
380	0.000039	0.027	0.000589	1.001
390	.000120	0.082	.002209	3.755
400	.000396	0.270	.009290	15.793
410	.001210	0.826	.034840	59.228
420	.004000	2.732	.096600	164.220
430	.011600	7.923	.199800	339.660
440	.023000	15.709	.328100	557.770
450	.038000	25.954	.455000	773.500
460	.060000	40.980	.567000	963.900
470	.090980	62.139	.676000	1149.200
480	.139020	94.951	.793000	1348.100
490	.208020	142.078	.904000	1536.800
500	.323000	220.609	.982000	1669.400
507	.444310	303.464	1.000000	1700.000
510	.503000	343.549	.997000	1694.900
520	.710000	484.930	.935000	1589.500
530	.862000	588.746	.811000	1378.700
540	.954000	651.582	.650000	1105.000
550	.994950	679.551	.481000	817.700
555	1.000000	683.000	.402000	683.000
560	.995000	679.585	.328800	558.960
570	.952000	650.216	.207600	352.920
580	.870000	594.210	.121200	206.040
590	.757000	517.031	.065500	111.350
600	.631000	430.973	.033150	56.355
610	.503000	343.549	.015930	27.081
620	.381000	260.223	.007370	12.529
630	.265000	180.995	.003335	5.670
640	.175000	119.525	.001497	2.545
650	.107000	73.081	.000677	1.151
660	.061000	41.663	.000313	0.532
670	.032000	21.856	.000148	0.252
680	.017000	11.611	.000072	0.122
690	.008210	5.607	.000035	.060
700	.004102	2.802	.000018	.030
710	.002091	1.428	.000009	.016
720	.001047	0.715	.000005	.008
730	.000520	0.355	.000003	.004
740	.000249	0.170	.000001	.002
750	.000120	0.082	.000001	.001
760	.000060	0.041		
770	.000030	0.020		

Irradiance and Illuminance:

Irradiance is a measure of radiometric flux per unit area, or flux density. Irradiance is typically expressed in W/cm^2 (watts per square centimeter) or W/m^2 (watts per square meter).

Illuminance is a measure of photometric flux per unit area, or visible flux density. Illuminance is typically expressed in lux (lumens per square meter) or foot-candles (lumens per square foot).

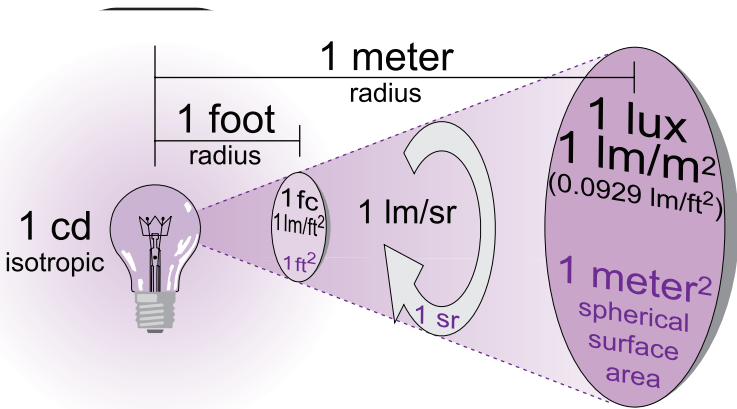


Fig. 7.4 Irradiance.

In figure 7.4, above, the light bulb is producing 1 candela. The candela is the base unit in light measurement, and is defined as follows: a 1 candela light source emits 1 lumen per steradian in all directions (isotropically). A steradian is defined as the solid angle which, having its vertex at the center of the sphere, cuts off an area equal to the square of its radius. The number of steradians in a beam is equal to the projected area divided by the square of the distance.

So, 1 steradian has a projected area of 1 square meter at a distance of 1 meter. Therefore, a 1 candela (1 lm/sr) light source will similarly produce 1 lumen per square foot at a distance of 1 foot, and 1 lumen per square meter at 1 meter. Note that as the beam of light projects farther from the source, it expands, becoming less dense. In fig. 7.4, for example, the light expanded from $1 \text{ lm}/\text{ft}^2$ at 1 foot to $0.0929 \text{ lm}/\text{ft}^2$ (1 lux) at 3.28 feet (1 m).

Cosine Law

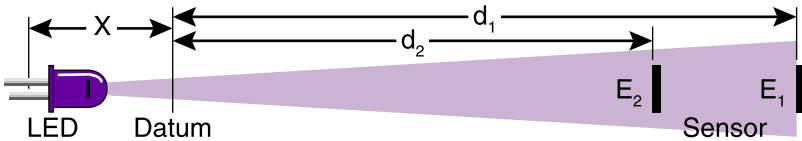
Irradiance measurements should be made facing the source, if possible. The irradiance will vary with respect to the cosine of the angle between the optical axis and the normal to the detector.

Calculating Source Distance

Lenses will distort the position of a point source. You can solve for the virtual origin of a source by measuring irradiance at two points and solving for the offset distance, X , using the Inverse Square Law:

$$E_1(d_1 + X)^2 = E_2(d_2 + X)^2$$

Figure 7.5 illustrates a typical setup to determine the location of an LED's virtual point source (which is behind the LED due to the built-in lens). Two irradiance measurements at known distances from a reference point are all that is needed to calculate the offset to the virtual point source.



$$X = [d_1(E_1 / E_2)^{1/2} - d_2] \div [1 - (E_1 / E_2)^{1/2}]$$

Fig. 7.5 Sometimes you have to solve for the true distance from source to sensor.

Units Conversion: Flux Density

IRRADIANCE:

1 W/cm² (watts per square centimeter)
= 10⁴ W/m² (watts per square meter)
= 6.83 x 10⁶ lux at 555 nm
= 14.33 gram*calories/cm²/minute

ILLUMINANCE:

1 lm/m² (lumens per square meter)
= 1 lux (lx)
= 10⁻⁴ lm/cm²
= 10⁻⁴ phot (ph)
= 9.290 x 10⁻² lm/ft²
= 9.290 x 10⁻² foot-candles (fc)

Radiance and Luminance:

Radiance is a measure of the flux density per unit solid viewing angle, expressed in $\text{W}/\text{cm}^2/\text{sr}$. Radiance is independent of distance for an extended area source, because the sampled area increases with distance, cancelling inverse square losses.

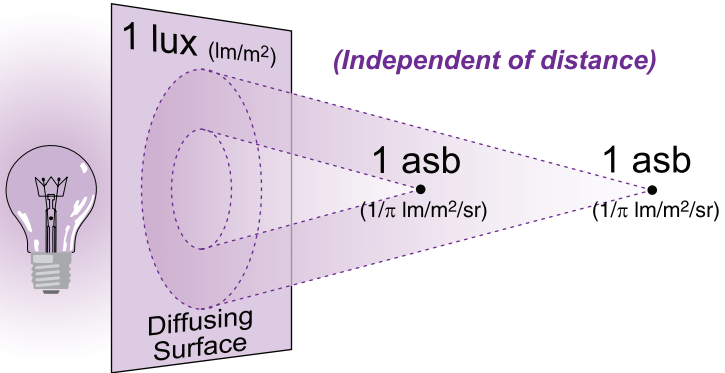


Fig. 7.6 Radiance

The radiance, L , of a diffuse (Lambertian) surface is related to the radiant exitance (flux density), M , of a surface by the relationship:

$$L = M / \pi$$

Some luminance units (asb, L , fL) already contain π in the denominator, allowing simpler conversion to illuminance units.

Example: Suppose a diffuse surface with a reflectivity, ρ , of 85% is exposed to an illuminance, E , of 100.0 lux (lm/m^2) at the plane of the surface. What would be the luminance, L , of that surface, in cd/m^2 ?

Solution:

1.) Calculate the luminous exitance of the surface:

$$M = E * \rho$$

$$M = 100.0 * 0.85 = \mathbf{85.0 \text{ lm}/\text{m}^2}$$

2.) Calculate the luminance of the surface:

$$L = M / \pi$$

$$L = 85.0 / \pi = 27.1 \text{ lm}/\text{m}^2/\text{sr} = \mathbf{27.1 \text{ cd}/\text{m}^2}$$

Irradiance From An Extended Source:

The irradiance, E , at any distance from a uniform extended area source, is related to the radiance, L , of the source by the following relationship, which depends only on the subtended central viewing angle, θ , of the radiance detector:

$$E = \pi L \sin^2(\theta/2)$$

So, for an extended source with a radiance of $1 \text{ W/cm}^2/\text{sr}$, and a detector with a viewing angle of 3° , the irradiance at any distance would be $2.15 \times 10^{-3} \text{ W/cm}^2$. This assumes, of course, that the source extends beyond the viewing angle of the detector input optics.

Units Conversion: Radiance & Luminance

RADIANCE:

$$\begin{aligned} 1 \text{ W/cm}^2/\text{sr} \text{ (watts per sq. cm per steradian)} \\ &= 6.83 \times 10^6 \text{ lm/m}^2/\text{sr} \text{ at } 555 \text{ nm} \\ &= 683 \text{ cd/cm}^2 \text{ at } 555 \text{ nm} \end{aligned}$$

LUMINANCE:

$$\begin{aligned} 1 \text{ lm/m}^2/\text{sr} \text{ (lumens per sq. meter per steradian)} \\ &= 1 \text{ candela/m}^2 \text{ (cd/m}^2\text{)} \\ &= 1 \text{ nit} \\ &= 10^{-4} \text{ lm/cm}^2/\text{sr} \\ &= 10^{-4} \text{ cd/cm}^2 \\ &= 10^{-4} \text{ stilb (sb)} \\ &= 9.290 \times 10^{-2} \text{ cd/ft}^2 \\ &= 9.290 \times 10^{-2} \text{ lm/ft}^2/\text{sr} \\ &= \pi \text{ apostilbs (asb)} \\ &= \pi \text{ cd}/\pi\text{/m}^2 \\ &= \pi \times 10^{-4} \text{ lamberts (L)} \\ &= \pi \times 10^{-4} \text{ cd}/\pi\text{/cm}^2 \\ &= 2.919 \times 10^{-1} \text{ foot-lamberts (fL)} \\ &= 2.919 \times 10^{-1} \text{ lm}/\pi\text{/ft}^2/\text{sr} \end{aligned}$$

Radiant and Luminous Intensity:

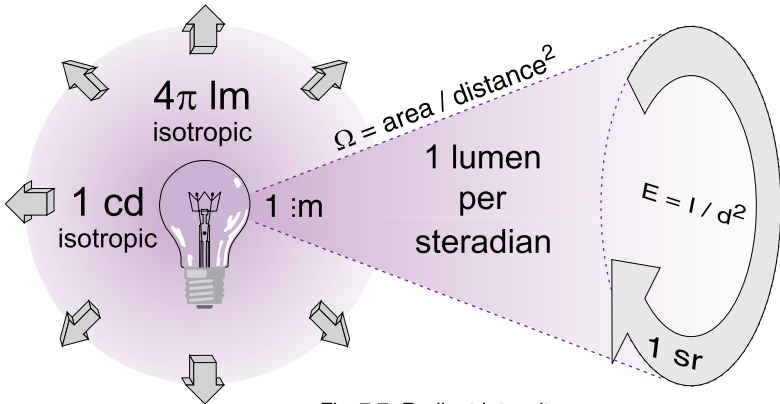


Fig. 7.7 Radiant Intensity.

Radiant Intensity is a measure of radiometric power per unit solid angle, expressed in watts per steradian. Similarly, luminous intensity is a measure of visible power per solid angle, expressed in candela (lumens per steradian). Intensity is related to irradiance by the inverse square law, shown below in an alternate form:

$$I = E * d^2$$

If you are wondering how the units cancel to get *flux/sr* from *flux/area* times *distance squared*, remember that steradians are a dimensionless quantity. Since the solid angle equals the area divided by the square of the radius, $d^2 = A / \Omega$, and substitution yields:

$$I = E * A / \Omega$$

The biggest source of confusion regarding intensity measurements involves the difference between Mean Spherical Candela and Beam Candela, both of which use the candela unit (lumens per steradian). Mean spherical measurements are made in an integrating sphere, and represent the total output in lumens divided by $4\pi \text{ sr}$ in a sphere. Thus, a one candela isotropic lamp produces one lumen per steradian.

Beam candela, on the other hand, samples a very narrow angle and is only representative of the lumens per steradian at the peak intensity of the beam. This measurement is frequently misleading, since the sampling angle need not be defined.

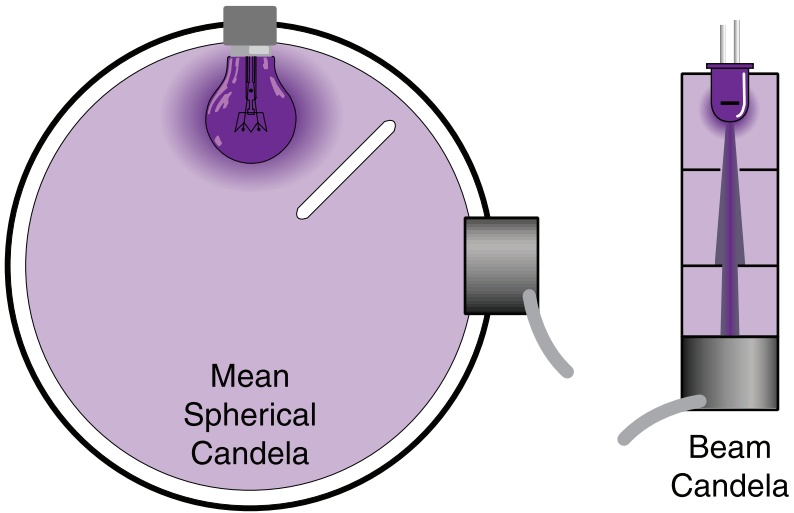


Fig. 7.8 Both mean spherical candela and beam candela are expressed in cd.

Suppose that two LED's each emit 0.1 lm total in a narrow beam: One has a 10° solid angle and the other a 5° angle. The 10° LED has an intensity of 4.2 cd, and the 5° LED an intensity of 16.7 cd. They both output the same total amount of light, however - 0.1 lm.

A flashlight with a million candela beam sounds very bright, but if its beam is only as wide as a laser beam, then it won't be of much use. Be wary of specifications given in beam candela, because they often misrepresent the total output power of a lamp.

Units Conversion: Intensity

RADIANT INTENSITY:

1 W/sr (watts per steradian)
 = 12.566 watts (isotropic)
 = $4 \cdot \pi$ W
 = 683 candela at 555 nm

LUMINOUS INTENSITY:

1 lm/sr (lumens per steradian)
 = 1 candela (cd)
 = $4 \cdot \pi$ lumens (isotropic)
 = 1.464×10^{-3} watts/sr at 555 nm

Converting Between Geometries

Converting between geometry-based measurement units is difficult, and should only be attempted when it is impossible to measure in the actual desired units. You must be aware of what each of the measurement geometries implicitly assumes before you can convert. The example below shows the conversion between lux (lumens per square meter) and lumens.

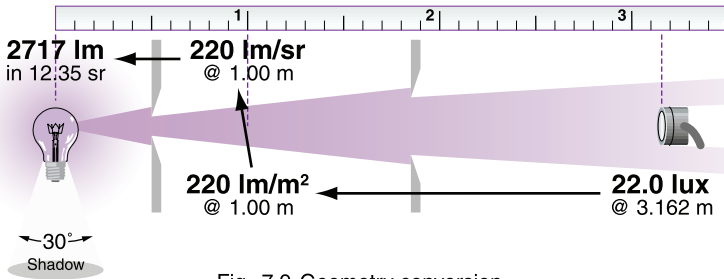


Fig. 7.9 Geometry conversion.

Example: You measure 22.0 lux from a light bulb at a distance of 3.162 meters. How much light, in lumens, is the bulb producing? Assume that the clear enveloped lamp is an isotropic point source, with the exception that the base blocks a 30° solid angle.

Solution:

1.) Calculate the irradiance at 1.0 meter:

$$E_1 = (d_2 / d_1)^2 * E_2$$

$$E_{1.0m} = (3.162 / 1.0)^2 * 22.0 = \mathbf{220 \text{ lm/m}^2}$$

2.) Convert from lm/m^2 to lm/sr at 1.0 m:

$$220 \text{ lm/m}^2 * 1 \text{ m}^2/\text{sr} = \mathbf{220 \text{ lm/sr}}$$

3.) Calculate the solid angle of the lamp:

$$\Omega = A / r^2 = 2\pi h / r = 2\pi[1 - \cos(\alpha / 2)]$$

$$\Omega = 2\pi[1 - \cos(330 / 2)] = \mathbf{12.35 \text{ sr}}$$

4.) Calculate the total lumen output:

$$220 \text{ lm/sr} * 12.35 \text{ sr} = \mathbf{2717 \text{ lm}}$$